



**AP<sup>®</sup> Calculus BC**  
**2007 Free-Response Questions**  
**Form B**

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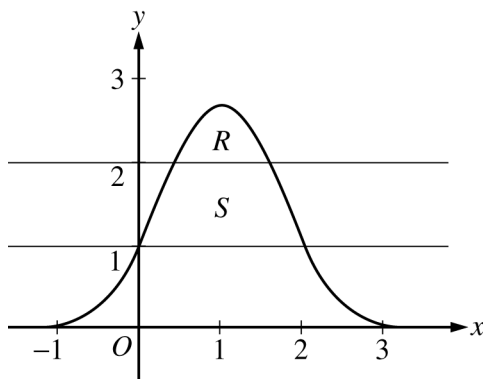
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**2007 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)**

**CALCULUS BC**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

A graphing calculator is required for some problems or parts of problems.

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1. Let  $R$  be the region bounded by the graph of  $y = e^{2x-x^2}$  and the horizontal line  $y = 2$ , and let  $S$  be the region bounded by the graph of  $y = e^{2x-x^2}$  and the horizontal lines  $y = 1$  and  $y = 2$ , as shown above.
- (a) Find the area of  $R$ .
- (b) Find the area of  $S$ .
- (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 1$ .
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**WRITE ALL WORK IN THE EXAM BOOKLET.**

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2. An object moving along a curve in the  $xy$ -plane is at position  $(x(t), y(t))$  at time  $t$  with

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right) \text{ and } \frac{dy}{dt} = \ln(t^2 + 1)$$

for  $t \geq 0$ . At time  $t = 0$ , the object is at position  $(-3, -4)$ . (Note:  $\tan^{-1}x = \arctan x$ )

- (a) Find the speed of the object at time  $t = 4$ .
  - (b) Find the total distance traveled by the object over the time interval  $0 \leq t \leq 4$ .
  - (c) Find  $x(4)$ .
  - (d) For  $t > 0$ , there is a point on the curve where the line tangent to the curve has slope 2. At what time  $t$  is the object at this point? Find the acceleration vector at this point.
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3. The wind chill is the temperature, in degrees Fahrenheit ( $^{\circ}\text{F}$ ), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity  $v$ , in miles per hour (mph). If the air temperature is  $32^{\circ}\text{F}$ , then the wind chill is given by  $W(v) = 55.6 - 22.1v^{0.16}$  and is valid for  $5 \leq v \leq 60$ .
- (a) Find  $W'(20)$ . Using correct units, explain the meaning of  $W'(20)$  in terms of the wind chill.
- (b) Find the average rate of change of  $W$  over the interval  $5 \leq v \leq 60$ . Find the value of  $v$  at which the instantaneous rate of change of  $W$  is equal to the average rate of change of  $W$  over the interval  $5 \leq v \leq 60$ .
- (c) Over the time interval  $0 \leq t \leq 4$  hours, the air temperature is a constant  $32^{\circ}\text{F}$ . At time  $t = 0$ , the wind velocity is  $v = 20$  mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at  $t = 3$  hours? Indicate units of measure.
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**END OF PART A OF SECTION II**

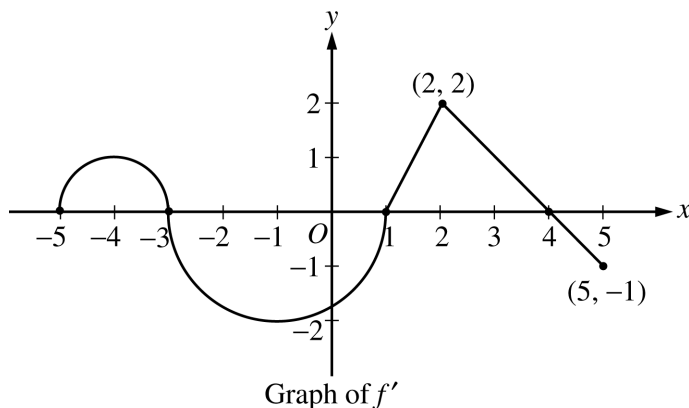
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CALCULUS BC  
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



4. Let  $f$  be a function defined on the closed interval  $-5 \leq x \leq 5$  with  $f(1) = 3$ . The graph of  $f'$ , the derivative of  $f$ , consists of two semicircles and two line segments, as shown above.
- For  $-5 < x < 5$ , find all values  $x$  at which  $f$  has a relative maximum. Justify your answer.
  - For  $-5 < x < 5$ , find all values  $x$  at which the graph of  $f$  has a point of inflection. Justify your answer.
  - Find all intervals on which the graph of  $f$  is concave up and also has positive slope. Explain your reasoning.
  - Find the absolute minimum value of  $f(x)$  over the closed interval  $-5 \leq x \leq 5$ . Explain your reasoning.

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5. Consider the differential equation  $\frac{dy}{dx} = 3x + 2y + 1$ .

(a) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ .

(b) Find the values of the constants  $m$ ,  $b$ , and  $r$  for which  $y = mx + b + e^{rx}$  is a solution to the differential equation.

(c) Let  $y = f(x)$  be a particular solution to the differential equation with the initial condition  $f(0) = -2$ . Use Euler's method, starting at  $x = 0$  with a step size of  $\frac{1}{2}$ , to approximate  $f(1)$ . Show the work that leads to your answer.

(d) Let  $y = g(x)$  be another solution to the differential equation with the initial condition  $g(0) = k$ , where  $k$  is a constant. Euler's method, starting at  $x = 0$  with a step size of 1, gives the approximation  $g(1) \approx 0$ . Find the value of  $k$ .

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6. Let  $f$  be the function given by  $f(x) = 6e^{-x/3}$  for all  $x$ .

(a) Find the first four nonzero terms and the general term for the Taylor series for  $f$  about  $x = 0$ .

(b) Let  $g$  be the function given by  $g(x) = \int_0^x f(t) dt$ . Find the first four nonzero terms and the general term for the Taylor series for  $g$  about  $x = 0$ .

(c) The function  $h$  satisfies  $h(x) = kf'(ax)$  for all  $x$ , where  $a$  and  $k$  are constants. The Taylor series for  $h$  about  $x = 0$  is given by

$$h(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

Find the values of  $a$  and  $k$ .

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**END OF EXAM**

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